An Axiomatic and Elicitation Perspective on Market Makers for Decentralized Exchanges

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Abstract

We introduce axioms for general asset market making, and apply them to study automated maker makers for decentralized exchanges. Our first result is a characterization of Constant-Function Market Makers (CFMMs) without transaction fees. We then give a general conceptual bridge between asset market making and prediction markets for ratios of expectations. As a special case, we derive a precise equivalence between CFMMs and cost-function market makers from the prediction markets literature.

Background

Decentralized exchanges use CFMM which enable a trade by an agent if the value of a pre-specified function φ remains the same before and after the trade.

Example - Uniswap allows a trade $r = (r_1, r_2)$ if $\varphi(r) = r_1 \cdot r_2 = k$.

Market Model

Asset market trades in n assets.

Trade $\mathbf{r} \in \mathbb{R}^n \implies$ trader sells r_i units of asset i if $r_i > 0$ else buys.

- At time t, history of trades be $h_{t-1} = (\mathbf{r}_0, \mathbf{r}_1, \ldots, \mathbf{r}_{t-1})$.
- Reserves the market maker has is $\mathbf{q}_{t-1} = \mathbf{r}_0 + \cdots + \mathbf{r}_{t-1}$.
- Out of available trades given at this time, represented by ValTrades(h_{t-1}), trader selects a trade $\mathbf{r}_t$.
- Market updates history and reserves $\mathbf{q}_t = \mathbf{q}_{t-1} + \mathbf{r}_t$, $h_t = h_{t-1} \oplus \mathbf{r}_t$.

Axiomatic Characterization

Shouldn't any market maker satisfy some basic properties stated below?

- **NoDominatedTrades** : A market maker should not offer strictly better/worse trades. This is justified by rationality of market maker and the traders.
- **PathIndependence** : Performing trade r followed by trade r' should be same as executing trade $r + r'$.
- **Liquidation** : If the trader comes to the market maker with a bundle $r \in \mathbb{R}^n_{\geq 0}$ and requests a bundle $r' \in \mathbb{R}^n_{\geq 0}$, $\exists \beta > 0$ such that maker maker accepts r for $\beta r'$.
- **DemandResponsiveness** : If a market allowed trade of assets $r \in \mathbb{R}^n_{\geq 0}$ for a bundle $r' \in \mathbb{R}^n_{\geq 0}$, then the "exchange rate" for these goods should increase for next trade. This enables the market maker to adapt "price" to reflect the demand.

Note that NoDominatedTrades and PathIndependence gives one no scope for arbitrage opportunities.

Theorem

A market satisfies StrongLiquidation, NoDominatedTrades, PathIndependence, *and* DemandResponsiveness *if and only if it is implemented using a CFMM with an increasing, concave function φ.*

While most CFMMs satisfy all the above axioms, our key result is that they are the only way to make these axioms true.

Can CFMMs and Cost-function market makers be the same?

Existing literature of Abernethy et. al 2013 and Frongillo & Waggoner 2018 characterize prediction markets for Arrow-Debreu securities to implement a cost-function market maker that sells securities $\mathbf{r}$ for $C(-\mathbf{q} - \mathbf{r}) - C(-\mathbf{q})$ cash where $-\mathbf{q}$ is total shares sold.

- Prediction markets trade securities and are designed to elicit forecasts of future events. They allow any bundle to be bought/sold.
- CFMMs are designed to provide liquidity and facilitate trades. They allow only certain bundles to be traded.

Theorem

In a strong sense, CFMMs and cost- function market makers are equivalent (i.e. have same available trades for a given history). One can create equivalent cost-function market maker if we know the CFMM and vice versa by the following maps -

$\psi_1 : \varphi \mapsto$ *Cost, where* $C(-\mathbf{q}) := c$ *s.t.* $\varphi(c\mathbb{1} + \mathbf{q}) = \varphi(\mathbf{r}_0)$

$\psi_2 :$ *Cost* $\mapsto \varphi$, *where* $\varphi(\mathbf{q}) := -C(-\mathbf{q})$.

For example : The cost-function market maker equivalent to uniswap ($\varphi(r) = r_1 \cdot r_2 = k$) is given by - $C(\mathbf{r}) = \frac{1}{2}\left(r_1 + r_2 + \sqrt{(r_1 - r_2)^2 + 4 r_{01} r_{02}}\right)$. where r_{01} is first co-coordinate of $\mathbf{r}_0$. This cost-function can be derived from the scoring rule $G(p) = 2\sqrt{k \cdot p \cdot (1-p)}$, which is square-root based scoring rule that appears in Buja et. al 2015. [1]

Proof idea :

- Cost-function market makers can be thought of as constant-risk market makers.
- Given ValTrades$_\varphi$, construct a convex risk measure by asking how much of the "grand bundle" (one unit of each asset) to add to a given bundle before the net trade would be allowed.

Future Work

- Creating CFMMs with adaptive liquidity.
- Impact of transaction fee; best way to impose the fee.

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